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Lattice study for conformal windows of SU(2) and SU(3) gauge theories with fundamental fermions*

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We present our investigation of SU(2) gauge theory with 8 flavours, and SU(3) gauge theory with 12 flavours. For the SU(2) case, at strong bare coupling, $\beta \lesssim 1.45$, the distribution of the lowest eigenvalue of the Dirac operator can be described by chiral random matrix theory for the Gaussian symplectic ensemble. Our preliminary result indicates that the chiral phase transition in this theory is of bulk nature. For the SU(3) theory, we use high-precision lattice data to perform the step-scaling study of the coupling, $g_{\rm GF}$, in the Gradient Flow scheme. We carefully examine the reliability of the continuum extrapolation in the analysis, and conclude that the scaling behaviour of this SU(3) theory is not governed by possible infrared conformality at $g^2_{\rm GF} \lesssim 6$.

The 33rd International Symposium on Lattice Field Theory
14 -18 July 2015
Kobe International Conference Center, Kobe, Japan

*Report numbers:HUPD-1507, LLNL-PROC-678652.
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1. Introduction

The search for infrared (IR) conformality in various gauge theories has recently been a popular subject in Lattice Field Theory [1, 2]. Results of such research activities can lead to useful information for constructing composite Higgs, or walking technicolour (WTC), models. These models contain dynamical electroweak symmetry breaking, and have been shown to be compatible with experiments at the LHC [3, 4]. Given a gauge group and a representation of the fermions, it is essential to determine the smallest number of flavours, $N_f^{\rm cr}$, where the theory can contain an infrared fixed point (IRFP). The value of $N_f^{\rm cr}$ is often referred to as the lower end of the conformal window for a family of theories. A viable WTC model can be obtained with the number of flavours just below $N_f^{\rm cr}$, such that the infrared (IR) nearly scale-invariant behaviour of the theory can produce a state that is parametrically light [5, 6].

In lattice determination of $N_f^{\rm cr}$, the need for accurate numerical calculations results from the challenge in distinguishing between IR conformality and slow-running behaviour in theories near the lower end of the conformal window. One example is the 12-flavour SU(3) gauge theory that has been studied by many groups. The majority concluded that the theory is IR conformal. Amongst these investigations, one popular strategy is the step-scaling method for computing the running coupling. This method was implemented using the Schrödinger Functional (SF) scheme [7] and the Twisted Polyakov Loop (TPL) scheme [8, 9], with a variation of it carried out for the Gradient Flow (GF) scheme [10]. All these previous studies lead to evidence for IR scale-invariance. Nevertheless, the smallness of the β-function in this theory has to be noted. Two-loop perturbation theory predicts at most 6% change of the renormalised coupling between the Gaussian ultraviolet (UV) and the possible strongly-coupled IR fixed points at doubling the length scale. Therefore, to make any statistically-meaningful statement regarding the running behaviour, it is desirable to have results with subpercentage-level error. Such precision was not achieved in the previous works, leaving room to improve the calculations. In this article we present our result of the step-scaling study for the GF-scheme coupling in this theory. We obtain the renormalised coupling at $\lesssim 0.5\%$ statistical error, and use two discretisations which allow us to check the reliability of the continuum extrapolation.

In this presentation, we also show preliminary results from our investigation of the chiral phase transition in SU(2) gauge theory with 8 flavours. As pointed out in Ref. [11], the 6-flavour SU(2) gauge theory can be confining. This makes the study of the 8-flavour theory interesting for determining $N_f^{\rm cr}$ for the case of the SU(2) gauge group. This theory may be IR conformal or confining with a very small β-function as well. There exist relatively few results concerning its IR behaviour [12, 13], and our investigation can lead to further information in this regard. In particular, we compute the distribution of the lowest eigenvalue of the Dirac operator, and compare with the prediction from the Random Matrix Theory (RMT). This method can be used to extract the infinite-volume chiral condensate from finite-volume lattice data, and there is no contamination from power divergences. Therefore it leads to more reliable results. In addition to its application in QCD, this approach has also been adopted for lattice computations for beyond the standard model physics in Ref. [14]. Using this strategy, we find the existence of the chirally broken phase in SU(2) gauge theory with 8 flavours. Our preliminary results show that the chiral phase transition is of bulk nature.

2. Random Matrix Theory and SU(2) gauge theory with eight flavours

We use the plaquette gauge action and unimproved staggered fermions in the simulation. The lattice volumes, $\hat{V} = \hat{T} \times \hat{L}^3$, are $\hat{T} = \hat{L} = 8$, 12 and 16. The fermion masses are $\hat{m}_f = am_f = 0.005$, 0.010 and 0.015. We study the phase structure of this system by computing the chiral condensate. Investigation of the plaquette and Polyakov loop of this theory was reported in [15]. We use chiral RMT to extract the chiral condensate from our data. The RMT provides a reliable procedure which is free from power divergences and finite volume effects, if the system is in the ε-regime.

The dynamical variables for the RMT are the eigenvalues of an $N \times N$ matrix, where N should be taken $N \to \infty$. After suitable rescaling, the eigenvalues ζ_i follow the distribution:

$$\rho_N^{(\beta)}(\zeta_1, \ldots, \zeta_N; \mu_1, \ldots, \mu_{N_f}) = C \prod_{i=1}^{N} \left(\zeta_i^{\beta(\nu+1)-1} e^{-\beta \frac{\zeta_i^2}{8N}} \prod_{a=1}^{N_f} (\zeta_i + \mu_a^2) \right) \prod_{i>j}^{N} |\zeta_i^2 - \zeta_j^2|^\beta, \tag{2.1}$$

where N_f is the number of flavours, μ_a are mass parameters, ν is the topological charge, and C is the overall normalisation. The Dyson index β is 4 for staggered fermions in the SU(2) fundamental representation [16]. As pointed out in [17], the corresponding number of flavours for RMT is different from that for the lattice simulation $N_f^{\text{lat.}}$:

$$N_f = 2 \cdot \frac{1}{4} \cdot N_f^{\text{lat.}} = 4, \tag{2.2}$$

where the factor 2 comes form the 2-fold degeneracy due to pseudo reality of the SU(2) gauge group, and $1/4$ is from the taste breaking of staggered fermion.

In this work, we use the distribution of the lowest eigenvalue derived from Eq. (2.1) [18]. The lowest eigenvalue, λ_1, of the massless Dirac operator computed on the lattice should follow the same distribution, after rescaling with the chiral condensate Σ, as predicted by the RMT:

$$p_1^{\text{RMT}}(\zeta_1; \mu)\Big|_{\zeta_1 = \lambda_1 V \Sigma, \mu = m_f V \Sigma} = \frac{1}{\hat{V}\hat{\Sigma}} p_1^{\text{latt.}}(\hat{\lambda}_1; \hat{m}_f), \tag{2.3}$$

where $\hat{\Sigma} = a^3 \Sigma$, $\hat{\lambda}_1 = a\lambda_1$, $V = a^4 \hat{V}$ and $m_f = \hat{m}_f / a$, with $\hat{V}$ and $\hat{m}_f$ being inputs in our lattice simulations. Equation (2.3) can be used to determine $\hat{\Sigma}$, given the $p_1^{\text{latt}}(\lambda_1; m_f)$ computed on the lattice, and the RMT-predicted $p_1^{\text{RMT}}(\zeta_1, \mu)$. Non-applicability of the RMT for extracting the condensate means the restoration of chiral symmetry.

The analytic expression of $p_1^{\text{RMT}}(\zeta_1, \mu)$ can be complicated. In fact, the available analytic result for the Gaussian symplectic ensemble is only applicable to cases of fractional topological charges [18]. For this reason, we use the Hybrid Monte Carlo (HMC) method to obtain the distribution with ζ_i treated as the dynamical variables in Eq. (2.1). We find that $N = 400$ is large enough, such that our results do not show significant N-dependence. Distributions with various values of μ from 0.0 to 100.0 are obtained by the HMC strategy, and then interpolated to $\mu = m_f V \Sigma$.

Two typical examples of the fits at $\nu = 0$ are shown in Fig. 1. The bad fit (right panel, $\beta = 1.475$) gives a very small value of Σ, indicating the restoration of chiral symmetry[1]. According to the previous study of this system [15], the theory at $\beta = 1.475$ is in the weak-coupling phase, while at $\beta = 1.3$ it is in the strong-coupling phase.

[1] We also observed that $N_f = 8$ and $N_f = 16$ RMT have a tendency to give poorer fitting, which is consistent with the taste breaking. Our data does not show clustering of the eigenvalues for the taste symmetry.

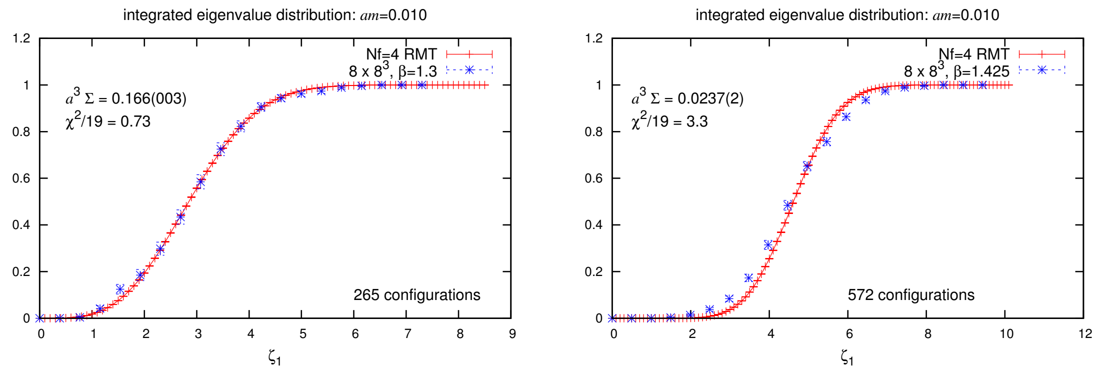

Figure 1: Examples of good fit (left) and bad fit (right) of the chiral condensate Σ by using the integrated distribution of lowest eigenvalue at $\nu = 0$. The red lines are from $N_f = 4$ RMT, and the blue crosses are the lattice data, with the eigenvalue, λ_1, rescaled to $\zeta_1 = V\Sigma\lambda_1$ using the value of Σ shown in the plot.

Figure 2 displays plots of Σ against β. All the panels show that the chiral condensate vanishes for $\beta \gtrsim 1.45$, and most of the fit results with small Σ come with poor $\chi^2/\text{d.o.f}$ (thin symbols). The location of this chiral phase transition is observed to have almost no volume (upper-left panel) and fermion-mass (other panels) dependence. This implies that the transition is of bulk nature, and the phase connecting to the weak-coupling continuum limit is the symmetric phase[2]. We are currently generating lattice data at larger volumes to further check the validity of this scenario. More detailed analysis of systematic errors, such as the effect of the taste breaking, is also needed.

3. The Yang-Mills gradient flow and SU(3) gauge theory with twelve flavours

For the study of SU(3) gauge theory with 12 flavours, we adopt the step-scaling method to investigate the renormalised running coupling, $g_{\rm GF}$, in the Gradient Flow (GF) scheme [19, 20]. In this work we use two discretisations, namely the clover and the plaquette, for extracting this coupling. This allows us to examine the reliability of our results in the continuum limit. To ensure that we only have one length scale for probing the theory, it is necessary to fix the ratio $c_\tau = \sqrt{8t}/L$, where L is the lattice size and t is the flow time. We implement the colour-twisted boundary condition, and perform the simulations at vanishing fermion mass. The work presented here has been reported in our recent paper, Ref. [21], which we refer to for more details and unexplained notation. Our goal is to compute, in the continuum limit,

$$r_\sigma(L) \equiv \frac{g^2_{\rm GF}(2L)}{g^2_{\rm GF}(L)}, \tag{3.1}$$

where L is interpreted as the renormalisation scale. To proceed, we first specify a value, u, and tune the bare couplings on the $\hat{L} \equiv L/a = 8, 10, 12$ lattices, such that the renormalised couplings, $\bar{g}^2_{\rm latt}$, extracted on these lattices all match this value. This u is interpreted as the continuum coupling, $g^2_{\rm GF}$, renormalised at L. The details of this tuning procedure is explained in Refs. [8, 21]. We then use

[2] Note that the continuum 8-flavour SU(2) theory is described by the RMT for the Gaussian orthogonal ensemble [16], and we are aware that this may complicate the interpretation of our results.

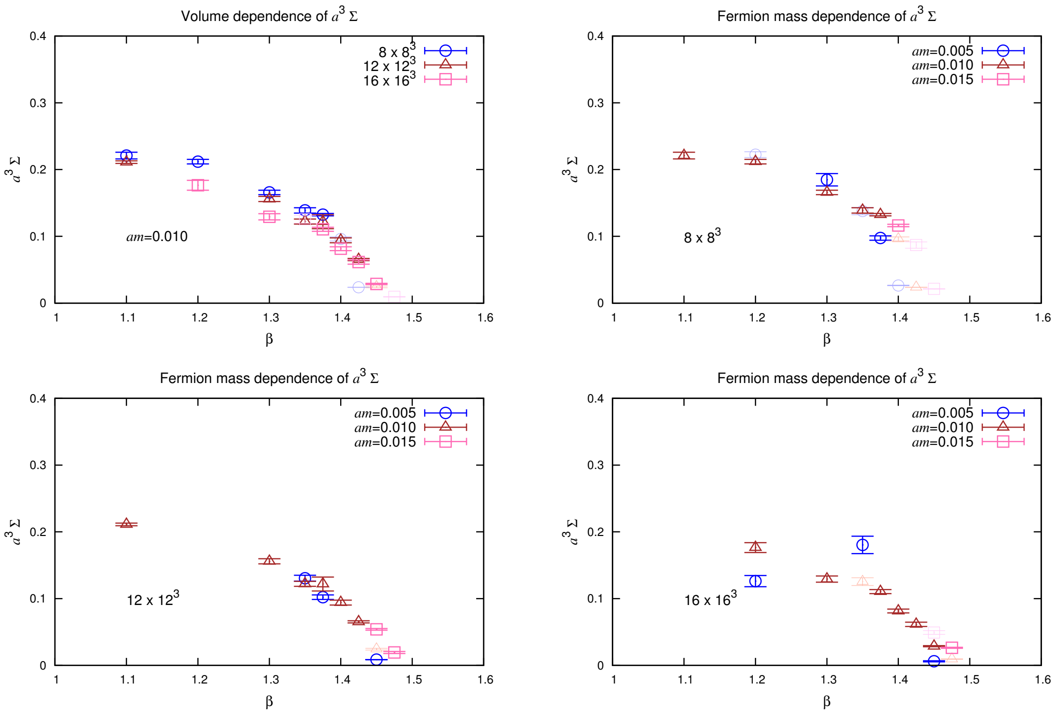

Figure 2: Chiral condensate Σ, extracted in the $\nu = 0$ sector, versus the bare coupling β. The upper left panel is with fermion mass $am_f = 0.010$ and several lattice volumes. The other plots are at fixed lattice volume with several fermion mass. Thin symbols indicate poor fitting of Σ with $\chi^2/\text{d.o.f} \geq 1$.

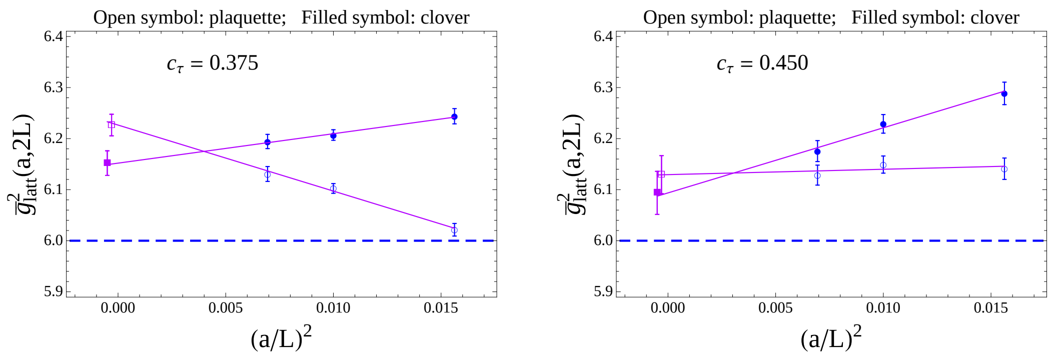

Figure 3: Continuum extrapolation for the step-scaling function for input $u = 6.0$ at $c_\tau = 0.375$ and 0.45.

the bare couplings obtained above to determine the lattice step-scaling functions, $\Sigma(u,a)$, which are simply $\bar{g}^2_{\rm latt}$ computed on the corresponding lattices $2\hat{L} = 16, 20, 24$. This allows us to perform the linear continuum extrapolation using the ansätz *a'la* Symanzik, $\Sigma(u,a) = \sigma(u) + A(a/L)^2$. The ratio, r_σ, defined in Eq. (3.1) can then be expressed as $r_\sigma(u) = \sigma(u)/u$.

Figure 3 shows the continuum extrapolation of the step-scaling function using the functional form linear in $(a/L)^2$, at $c_\tau = 0.375$ and 0.45, in the strong-coupling regime (input $g^2_{\rm GF} = 6$). These plots clearly demonstrate that one has to be careful when performing this extrapolation. As can be seen for the case of $c_\tau = 0.375$, the extrapolations are mild and smooth. On the other hand, the two

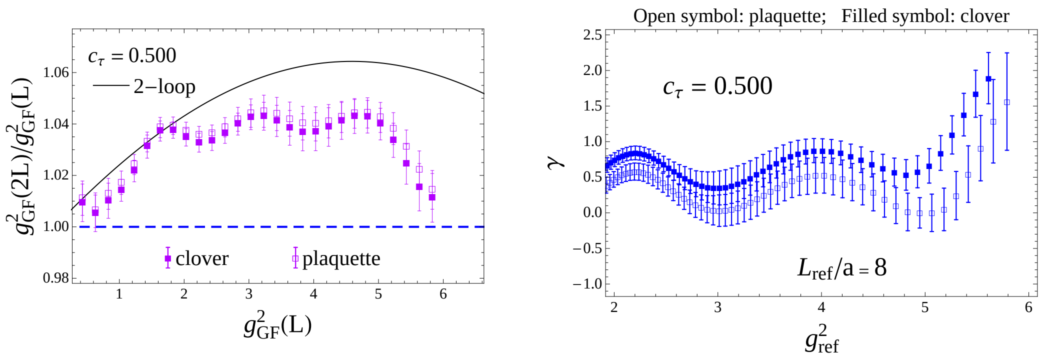

Figure 4: Left: Results of r_σ plotted against input $g^2_{\rm GF}(L)$ at $c_\tau = 0.5$. Right: Plot of γ against $g^2_{\rm ref}$.

discretisations do not lead to compatible result in the continuum limit, indicating that the effects of the lattice artefacts are significant. The situation is improved at increasing c_τ. In this work, we find that the continuum extrapolation is under control at $c_\tau \geq 0.45$.

Results of the ratio r_σ for $c_\tau = 0.5$ are displayed in the left panel Fig. 4. From this plot, we observe that at $g^2_{\rm GF} \gtrsim 5$, the GF-scheme coupling runs significantly slower than the two-loop perturbative prediction. At input $g^2_{\rm GF}(L) = 5.8$, r_σ is almost consistent with unity. However, one has to be cautious in using this result to indicate the existence of an IRFP in this theory. In Ref. [21], we argue that the continuum extrapolation of the lattice data for the coupling constant near an IRFP can be a subtle issue, and it is desirable to perform a finite-size scaling test. This test involves fitting the lattice numerical results for the (cutoff-dependent) renormalised coupling, $\bar{g}^2_{\rm latt}(g_0^2, \hat{L} = L/a)$, at various values of $\hat{L}$ to the formula

$$\bar{g}^2_{\rm latt}(g_0^2, \hat{L}) = g_l^2(g_{\rm ref}) + \left[g^2_{\rm ref} - g_l^2(g_{\rm ref})\right] \left(\frac{\hat{L}_{\rm ref}}{\hat{L}}\right)^{\gamma(g_{\rm ref})}, \quad (3.2)$$

where g_l and γ are free parameters, g_0 is the bare coupling, $g_{\rm ref} = \bar{g}_{\rm latt}(g_0^2, \hat{L} = \hat{L}_{\rm ref} = 8)$. This formula is derived from the "locally linearised" β-function. This linearisation is valid (away from the asymptotic-freedom limit) because the β-function is small. We implement the scaling test using Eq. (3.2) and scanning through many values of g_0. At each choice of g_0 (hence $g_{\rm ref}$ since we fix $\hat{L} = 8$), we perform a fit. When the bare coupling is tuned such that the theory is in the vicinity of the possible IRFP, g_l and γ will approach g_* (the location of the fixed point) and γ_* (the slope of the β-function at the zero in the strong-coupling regime). That is, the signal for the existence of IR conformality should be the plateaus in the plots of g_l and γ against $g_{\rm ref}$. In addition, scale invariance should ensure that results from different discretisations agree. The outcome of this analysis is shown in the right panel of Fig. 4. It is clear that our data do not indicate the existence of an IRFP in the regime where our studies are carried out.

4. Conclusion and outlook

In this talk, we present results from our lattice investigations for the IR behaviour of SU(2) gauge theory with 8 flavours, and SU(3) with 12 flavours. For the SU(2) case, our preliminary finding is that there exists a chirally broken phase that can be well described by the RMT of the Gaussian symplectic ensemble. We find evidence that the relevant chiral phase transition can be of bulk nature, and the theory in the continuum limit may be chirally symmetric. Presently we are performing simulations at larger volumes to obtain further information regarding this transition. As for the SU(3) theory, our study of the GF-scheme coupling shows that the behaviour theory is not governed by possible IR conformality at $g^2_{\rm GF} \lesssim 6$.

Acknowledgments We are indebted to Tatsumi Aoyama and Hideo Matsufuru for their help in developping the HMC code, and thank Luigi Del Debbio, Shinsuke M. Nishigaki, and Ben Svetitsky for helpful discussion. I.K. and C.-J.D.L. acknowledge research grant 102-2112-M-009-002-MY3 from Taiwanese MOST. The work of H.O. is supported by the RIKEN Special Postdoctoral Researcher program. The JSPS Grant-in-Aid for Scientific Research for Young Scientists (B) No.25800139 is also acknowledged by H.O.. E.R. acknowledges the support from the DOE under contract DE-AC52-07NA27344 (LLNL). Most of the computational work was performed at Taiwanese NCHC, and KMI (ϕ) at Nagoya University. C.-J.D.L. thanks the hospitality of CERN and Hiroshima University during the progress of these projects.

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